CERME9

Proceedings of the Ninth Congress of the European Society for Research in Mathematics Education

Editors: Konrad Krainer and Naďa Vondrová

Organized by: Charles University in Prague, Faculty of Education

Year: 2015

erme

european society for research in mathematics education

Editors: Konrad Krainer, Naďa Vondrová

Publisher: Charles University in Prague, Faculty of Education and ERME

Place: Prague, Czech Republic

Year: 2015

ISBN: 978-80-7290-844-8

Recommended citation:

Author: Title of the contribution. In K. Krainer & N. Vondrová (Eds.) (2015). *Proceedings of the Ninth Conference of the European Society for Research in Mathematics Education (CERME9, 4-8 February 2015)* (pp. xx-yy). Prague, Czech Republic: Charles University in Prague, Faculty of Education and ERME.

School mathematics and bureaucracy

David Kollosche

Universität Potsdam, Potsdam, Germany, david.kollosche@uni-potsdam.de

This essay focuses on the connections between calculation and bureaucracy and points out implications for mathematics education. A genealogic analysis which methodologically follows Nietzsche and Foucault is used to show these connections firstly on a historical and secondly on the level of styles of thought. In the process, Weber's theory of bureaucratic administration, Sybille Krämer's history of formalisation and Foucault's analysis of the modern episteme will be considered. This study shows that mathematics – even in its unapplied and theoretical form – and bureaucracy share a common style of thought. Consequently, school mathematics can be understood as an institution which trains and examines a bureaucratic style of thought – an understanding supported by Paul Dowling's sociology of mathematics education.

Keywords: Calculation, formalisation, bureaucracy, Foucault, genealogy.

FROM SKOVSMOSE'S QUESTION TO THE METHOD OF GENEALOGY

In mathematics education research, calculation tasks, i.e. tasks which can be solved following prescribed rules for the manipulation of terms of numbers and variables, are often considered to be too 'bureaucratic' and to be over-represented in the mathematics classrooms. Usually, this claim serves to promote innovative approaches in mathematics education, but within this study it has not been possible to find research with such an object of an academic study itself. Ole Skovsmose (2005) also refers to the dominance of calculation tasks to distinguish his *Critical Mathematics Education*. He tries to explain the social function of the vast amount of calculation tasks each student has to solve during his school career. He refuses some common explanations and eventually asks:

> Could it be that 'normal' students in fact learn 'something', although not strictly speaking mathematics, and that this 'something' serves an important social function? If we look back again at the 10,000 commandments [which students have to solve during their school careers], what do they look like? Certainly, not like any of those tasks with which applied mathematics occupies itself, tasks in which creativity is needed to construct a model of a selected piece of reality. Nor do they look like anything a working mathematician is doing. However, they might have some similarities with those routine tasks, which are found everywhere in production and administration. An accountant has to do sums day after day. A laboratory assistant has to do a series of routine tasks in a careful way. [...] All such jobs do not invite creative ways of using numbers and figures. Instead things have to be handled carefully and correctly in a pre-described way. Could it be that the school mathematics tradition is a well functioning preparation for a majority of students who come to serve in such job-functions? (Skovsmose, 2005, pp. 11–12)

At a first glance, it seems surprising that Skovsmose does not provide any answers to his questions although he might be considered one of the founders of socially concerned research in mathematics education. But a closer look shows that he does not consider mathematics itself but only its application and teaching as socially critical. For Skovsmose, mathematics education is "undetermined", "without essence"; it can "serve a grand variety of social, political, and economic functions and interests" (Skovsmose, 2011, p. 2). Indeed, only under this assumption can he reasonably promote his emancipatory pedagogy. Then again, this very assumption hinders him from socially analysing non-applied calculation. So what if this assumption is misleading, if it is hindering us from discussing the social functions of what might belong to the essence of mathematics education: pure but nevertheless socially relevant mathematics, e.g. calculation? The question would then not only be how calculation *is* *applied*, but how it *can be applied*, which perspectives it allows and which ones it denies.

As calculation is no matter of applications and teaching alone, but one of mathematics itself, the usual methods of Critical Mathematics Education are not promising for a comparison of calculation and bureaucracy. Instead, this study uses the method of *genealogy* as introduced by Friedrich Nietzsche and continued by Michel Foucault. Genealogy is a method for "examining the historical origins of present day philosophical concepts, ideas and discourses along with the institutions that sprang from them" (Lightbody, 2010, p. 1). Unlike other historical approaches, it questions the ethical, metaphysical and epistemological values which implicitly underlie our knowledge and practices. It assumes that our ideas and practices are not born perfect, but have evolved in a struggle against other ideas and practices. Therefore, genealogy "is not the erecting of foundations: on the contrary, it disturbs what was previously considered immobile; it fragments what was thought unified" (Foucault, 1971/1984, § 3). By confronting the taken-for-granted with its fragile genesis, one gains the possibility of looking at it from an outsider's perspective and recognising the ideas it was directed against, the interests it served or the complicity into which it seduced us (Saar, 2007, p. 15). As every genealogy involves the danger of shaking our own convictions, it cannot be performed in any 'objective' way. Consequently, the purpose of genealogy is not to constitute any kind of 'true' history (although it might contribute to such an endeavour), but to help us understand our contemporary ideas and practices. Especially, it allows us to gain a critical distance from the values operating in mathematics education and to analyse calculation inside and outside school as a morally ambiguous socio-cultural phenomenon. Martin Saar states:

> Genealogy is more qualified than any other form of critique to grasp phenomena such as imperfect liberty, complicity with authority and subtle heteronomy, for it illuminates the conditions of the possibility of life forms in which heteronomy stabilises and power effects mentalities. (Saar, 2007, pp. 15–16; my translation)

The following analysis will first consider the genesis of calculation and bureaucracy in order to trace similarities in development. It will then match the styles of thought represented by calculation and bureaucracy in order to substantiate the claim that both share a common style of thought. The genealogic questions to ask are: *At what times, under which circumstances, to satisfy which needs and to serve whose interest did calculation and bureaucracy develop? What are the values underlying calculation and bureaucracy? What can the history of both tell us about commonalities between them?*

TOWARDS A GENEALOGY OF CALCULATION AND BUREAUCRACY

Unfortunately, neither a genealogy of calculation nor one of bureaucracy exists. Therefore, this study will try to sketch a genealogic approach from the theory that exists on the history and theory of calculation and bureaucracy. Contemporary theories of bureaucracy are still based on the sociology of Max Weber (Anter et al., 2010). Apart from that, his approach towards bureaucracy is particularly valuable for this study as it has genealogic features (Saar, 2007, p. 296). Weber describes an 'ideal type' of bureaucracy which is well summarised by Robert K. Merton:

> A formal, rationally organized social structure involves clearly defined patterns of activity in which, ideally, every series of actions is functionally related to the purposes of the organization. In such an organization there is integrated a series of offices, of hierarchized statuses, in which inhere a number of obligations and privileges closely defined by limited and specific rules. Each of these offices contains an area of imputed competence and responsibility. Authority, the power of control which derives from an acknowledged status, inheres in the office and not in the particular person who performs the official role. Official action ordinarily occurs within the framework of pre existing rules of the organization. The system of prescribed relations between the various offices involves a considerable degree of formality and clearly defined social distance between the occupants of these positions. (Merton, 1949, p. 151)

Although Weber considers the rise of bureaucracy a modern phenomenon and contrasts it with patrimonial forms of administration, which were typical for pre-modern monarchies, he acknowledges that some of its social and economic preconditions (such as the economic need for an effective, professional and centralised administration or the development of monetary economy) existed before, leading to historical forms of administrations with bureaucratic traits. He explicitly mentions the New Kingdom of Egypt, the late Roman Principate and the absolute monarchies of early modern Europe (Weber, 1922/1972, pp. 556, 560).

It is striking that our records of the development of calculation date back to the very same places and eras. In Sybille Krämer's unique history of formalisation (1988), which builds on the work of Jacob Klein (1936/1992), the outstanding contributors to that development of calculation from purely arithmetic to algebraic forms are the Egyptians at the beginning of the New Empire in the 16th cent. BC, Diophantus at the time of the late Roman Principate and Vieta when monarchy began its change towards absolutism. In spite of their enormous contributions to philosophy, the (decentrally administered) ancient Greeks considered calculation unworthy of a scientific discussion. It took only 200 years and the uprising of strict philosophical logic to have many contributions of the Pythagoreans, mathematicians said to be influenced by the Orient, excluded from the corpus of mathematics, most notably from Euclid's *Elements* (Krämer, 1988).

In a collection of application tasks and solutions, the Egyptians documented their mathematical techniques which include fractions, written methods for multiplication and division, applications of the Pythagorean theorem, solving quadratic equations and calculating areas and volumes. The Egyptian 'aha-calculus' is the earliest record of the use of variables; it documents the transfer of algorithms from numbers to signs. Different from our use today, the Egyptian variable could only stand for a specific, yet unknown number. It could be used in expressions such as $4 + h = 15$, but not in expressions which describe relations of values such as $y = 4x - 1$ or $a + b = b + a$ (although we have to keep in mind that these expressions could only be recorded verbally as our formalistic writing of mathematics developed only during the last few centuries). The variable was always connected to a certain number, it was its placeholder; and initially this was the only reason to treat it as a number (Krämer, 1988). Greek algebra separated values from their contexts of application and linked them to geometry. While expressions such as $a + b = b + a$ could now be interpreted as an apposition of line segments, the use of algebra and variables was constrained by the necessity of its geometrical interpretation. Five centuries after Euclid, Diophantus emerged as the *enfant terrible* of classical mathematics: he added lengths and areas, thought of triangles as triples of numbers, introduced symbols for operations and facilitated a formal notation for terms and equations. Nevertheless, he still considered variables the mere placeholder of a fixed number. Thus, he was unable to present universal algorithms and had to document his techniques in examples of tasks and solutions (Krämer, 1988; Klein, 1936/1992). This did not change until Vieta developed his algebra. Vieta was the first to consider variable as autonomous entities, independent from any number(s) it might represent and defined only by its rules of calculation:

> Algebra is no longer calculation with unknown numbers. Instead, it can be conceived as a calculation with characters, i.e. with 'undetermined' symbols which can represent all possible numbers that – substituted into a given equation – form a right expression [...] This is how the mathematical formula came into the world. (Krämer, 1988, p. 61; my translation)

A COMMON STYLE OF THOUGHT

The joint development of calculation and bureaucracy merely indicates a connection between both. The further analysis will show that calculation and bureaucracy do not only share a common style of 'bureaucratic thought', but that this style of thought is exemplary and prototypical for the modern thought since the 17th century. To this end, considerations about the characteristics of symbols in modern thought will lay the basis for analysing the role of bureaucratic thought in contemporary society.

Again, the genealogic analysis lays its focus on historical events of change and conflict. In this case, the biggest changes can be spotted around 1600, when Vieta's algebra became influential and the rise of bureaucracy allowed absolutism to develop. In his *Order of Things* (1966/1970), Michel Foucault identified a strong change in the episteme, i.e. the way people perceive and make sense of the world, in the years around 1600. Until the end of the Renaissance, thought was dominated by the principle of resemblance – a relationship considered to be unbreakable. Signs were "thought to have been placed upon things so that men might be able to uncover their secrets, their nature or their virtues" (p. 59). Signs resembled the represented, literature resembled truth, variables resembled numbers and money was made of valuable materials. In contrast to that, signs gain their independence in the 17th century. Suddenly, they are considered arbitrary constructs and require legitimisation. Consequently, science begins to discuss the criteria for the significance of symbols, leading to the appreciation of calculation and the evolution of formal logic. From then on, symbols are interrelated by their order (*taxonomy*, connected to logic) and by their measure (*mathesis*, connected to calculation) (pp. 71–76).

In his history of Algebra, Klein argues that while "in Greek science, concepts are formed in continual dependence on 'natural,' prescientific experience, from which the scientific concept is 'abstracted'", in modern science "nothing but the internal connection of all the concepts, their mutual relatedness, their subordination to the total edifice of science, determines for each of them a *univocal* sense". Klein recognised that "the nature of the modification which the mathematical science of the sixteenth and seventeenth century brings about [...] is *exemplary* for the total design of human knowledge in later times" (Klein, 1936/1992, pp. 120–121). From that perspective, modern calculation is not only an example of the new episteme, since it uses autonomous symbols; it is also a condition of the possibility of the modern episteme, for it constitutes a method to interrelate autonomous symbols. Accordingly, Klein points out that the new form of calculation is not a mere "device" of science but predefines the forms, e.g. the possibilities and restrictions, of scientific understanding (pp. 3–4).

The modern episteme is a prerequisite of bureaucracy, too, for it builds on the dissolution of the resemblance and the installation of symbolic practices. Bureaucracy has the purpose to provide predictable and equitable, i.e. non-arbitrary, forms of administration. For that reason, administrative acts are bound by "a consistent system of abstract rules which have normally been intentionally established" (Weber, 1921/1947, p. 330) instead of resembling any natural, traditional or divine law. Within this system of rules, officials act in a "spirit of formalistic impersonality", "without hatred or passion", "without affection or enthusiasm"; "everyone in the same empirical situation" has to be treated equally and the official is not allowed any "personal considerations" (p. 340). According to this, obligations, administrative means and authority are linked to positions, which are abstract symbols within the system of rules and do not resemble any natural person; positions are only 'held' by persons (p. 330). Bureaucracy follows the principle of impersonality by separating the official and the client from the human, by ignoring their individuality: their hope, fear, anger, gratitude, concern and doubt.

Calculation embodies a similar style of thought. Firstly, calculation is used for non-arbitrary, i.e. 'objective' predictions. The German *berechenbar* means 'calculable' as well as 'predictable'. Secondly, calculation works along a system of abstract rules that are culturally established and that the individual has to conform to. Thirdly, this system of rules demands 'formalistic impersonality' as calculation operates by its rules alone. This formalism disregards any 'personal considerations' of the calculating individual just as it disregards those of the official. But on top of this, the variables of each calculation also have to be manipulated 'impersonally', i.e. without any regard for what they might stand for. Every situation is only perceived in the boundaries of the pre-defined cases, i.e. cases that rules (for calculation or administration) exist for. It is this separation of sign and represented, of case and individual, of variable and number in the modern episteme that allows both bureaucracy and calculation as known today. Calculation is not a mere tool of bureaucratic administration, but it is in itself a technique for the "de-humanised" (Weber, 1922/1972, p. 563) processing of situations. Therefore, calculation is not an "undetermined" technique that can "serve a grand variety of social, political, and economic functions and interests" (Skovsmose, 2011, p. 2); it is a technique which resembles a style of thought that is: bureaucratic.

CALCULATION AND BUREAUCRACY IN THE CLASSROOM

Merton acknowledges that bureaucracy has to exert "a constant pressure upon the official to be methodical, prudent disciplined"; it must attain "an unusual degree of conformity with prescribed patterns of action" in order to fulfil its purpose (Merton, 1949, p. 154). Accordingly, Weber states that the bureaucratic style of thought requires "specialised training" (Weber, 1922/1972, p. 552). He explains the uprising of general education in modern times with the need for preparatory training and selection. Mathematics education in particular has historically developed alongside the cultivation of bureaucracy, incorporating calculation which has been shown to share a common style of thought. As Skovsmose points out, students have to solve a large amount of calculation tasks during their school career (Skovsmose, 2005). Solving these tasks is a prescribed activity with abstract symbols, following prescribed rules. Compared to other tasks used in school, calculation tasks specifically cannot be solved *without* a bureaucratic style of thought: There is usually no other valued solution to a calculation task than the development or application of a rule-bound and impersonal algorithm. The experience of these ever-repeating challenges causes the student to adapt. On the one hand, she may be able and willing to cultivate a bureaucratic style of thought. This would allow her to perform well (at least as long as mathematics education incorporates calculation to a large extent) and experience herself as a successful learner. On the other hand, she may be either unable or unwilling to cultivate a bureaucratic style of thought. This would leave her to ever-repeating failure in calculation tasks. In the case of such a trauma, the only adaptation securing the student's dignity is to escape from the humiliating situations. As a physical escape is not tolerated, it has to be performed mentally: those students 'learn' that mathematics is 'nothing for them'. Ideal-typically, this organisation of the mathematics classroom results in the production of either accomplices or avoiders of mathematics, ensuring that mathematical rule goes unquestioned and thereby contributing to the domination of humans by mathematics which Roland Fischer (1984) has warned against.

In his *Sociology of Mathematics Education* (1998), Paul Dowling describes "myths" about mathematics which are spread in the mathematics classroom. He is especially interested in so called 'real world problems' in which problems formulated in situations of the real world are interpreted and solved by calculation. While no specific applications of calculation will be discussed here, Dowling's analysis helps to understand who calculation is *per se* positioned in the mathematics classroom. The *myth of reference* is a mechanism which makes students think that calculation is a universal tool capable of solving any real world problem. Dowling states and exemplifies that many 'real world problems' build on situations which would not be solved mathematically in everyday life (Dowling, 1998, pp. 4–7). In addition to that, the mathematics classroom does not usually present any 'real world problems' that cannot be solved mathematically. Therefore, school mathematics provides experiences which foster the belief that mathematics can be reasonably applied to solve *any* problem of the real world. To the extent to which these 'real world problems' are based on or result in calculation, 'real world problems' function as a mechanism to install calculation as an omnipotent means of perceiving and handling our world.

Dowling's *myth of participation* refers to a mechanism which fosters the belief that students will need mathematics to succeed in their everyday life outside school (Dowling, 1998, pp. 7–11). 'Real world problems' are often student-oriented, i.e. they build on situations that are close to the experiences of students, although the mathematics involved would usually not be used to solve such problems in the real world. Their latent message is: *Look at these examples from your everyday life and see how calculation is needed to manage them!* That is how school mathematics provides experiences which foster the belief that the students need mathematics and especially calculation to cope with their everyday life outside school.

Both mechanisms bear the possibility of intensifying the experiences students have with calculation tasks. On the one hand, those succeeding in calculation may be happy to master the seemingly omnipotent and even privately relevant, de-humanised, rule-bound approach towards our world. On the other hand, those failing in calculation may explain their failure with their own incompetence rather than with insufficiencies of the calculation method as the latter is believed to be omnipotent. Nevertheless, they may believe that calculation is important for their life outside school. In the end, they might come to think that they lack the ability to handle the mathematics necessary for a fulfilled life and feel compelled to lay their trust in mathematical experts. Thus, a function (although not an intended goal) of school mathematics would be not only to separate the capable and willing from the unable and unwilling, but also to make the latter appreciate their subordination.

As bureaucracy and calculation share a common style of thought, performance in calculation indicates whether or not students are suited for administrative positions: whether or not they can reduce situations to cases and calculations, and whether or not they can handle these cases calculations according to imposed rules disregarding their personal thoughts and feelings; whether or not they can separate from themselves an administrative or calculative processor of rules. But school mathematics also educates in the sense that it produces situations in which students cultivate their relationships towards calculation. Whether or not, or rather: how far this experience affects the student's relationship towards bureaucracy, is hard to tell. It seems at least natural that the estrangement from calculation to some extent coheres with an estrangement from any practices sharing a similar style of thought, especially from bureaucratic ones. Therefore, school mathematics can be considered an institution which (alongside other functions) identifies and trains a calculatory-bureaucratic elite and teaches the rest to subordinate to the calculatory-bureaucratic administration of our society. And although this explanation deserves further theoretical and empirical elaboration, it can already serve as a first answer to Skovsmose's questions, contribute to the socio-philosophical discussion about the essence of mathematics, question the educational objectives which mathematics educators assign to school mathematics, explain why mathematics is such a polarising subject and shed light on anxiety and joy, motivation and estrangement in mathematics classrooms. Within the research community especially, it would also add a new dimension to socially concerned research in mathematics education and call for ways to deal with this social function of school mathematics.

REFERENCES

Anter, A., Bruhns, H., & Duran, P. (2010). Max Weber und die Bürokratie: Einleitung [Max Weber and Bureaucracy. An Introduction]. *Trivium: Revue Franco-Allemande des Sciences Humaines et Sociales*, (7).

Dowling, P. (1998). *The Sociology of Mathematics Education. Mathematical Myths, Pedagogic Texts*. London: Falmer.

Fischer, R. (1984). Unterricht als Prozeß der Befreiung vom Gegenstand: Visionen eines neuen Mathematikunterrichts [Education as the Process of the Liberation of the Subject Matter: Visions for a New Mathematics Education]. *Journal für Mathematik-Didaktik, 5*, 51–85.

Foucault, M. (1970). *The Order of Things: An Archaeology of the Human Sciences*. New York, NY: Pantheon (Original work published 1966).

Foucault, M. (1984). Nietzsche, Genealogy, History. In P. Rabinow (Ed.), *The Foucault Reader* (pp. 76–100). New York, NY: Pantheon.

Klein, J. (1992). *Greek Mathematical Thought and the Origin of Algebra*. New York, NY: Dover (Original work published 1936).

Krämer, S. (1988). *Symbolische Maschinen* [Symbolic Machines]. Darmstadt, Germany: Wissenschaftliche Buchgesellschaft.

Lightbody, B. (2010). *Philosophical Genealogy: An Epistemological Resonstruction of Nietzsche and Foucault's Genealogical Method. Volume I*. New York, NY: Peter Lang.

Merton, R. K. (1949). *Social Theory and Social Structure*. Glencoe, IL: Free Press.

Saar, M. (2007). *Genealogie als Kritik. Geschichte und Theorie des Subjekts nach Nietzsche und Foucault* [Genealogy for Critique. History and Theory of the Subject Following Nietsche and Foucault]. Frankfurt am Main, Germany: Campus.

Skovsmose, O. (2005). *Travelling Through Education: Uncertainty, Mathematics, Responsibility*. Rotterdam, The Netherlands: Sense.

Skovsmose, O. (2011). *An Invitation to Critical Mathematics Education*. Rotterdam, The Netherlands: Sense.

Weber, M. (1947). *The Theory of Social and Economic Organization*. New York, NY: Oxford University Press (Original work published 1921).

Weber, M. (1972). *Wirtschaft und Gesellschaft: Grundriss der verstehenden Soziologie* [Economy and Society]. Tübingen, Germany: Mohr.