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Editors

Marta Pytlak, Tim Rowland, Ewa Swoboda

University of Rzeszów, Poland

Editors

Marta Pytlak, University of Rzeszów, Poland

Tim Rowland, University of Cambridge, UK

Ewa Swoboda University of Rzeszów, Poland

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SOCIAL FUNCTIONS OF SCHOOL MATHEMATICS

David Kollosche

Universität Potsdam, Germany

In this essay, I explore the question which social functions school mathematics might hold. After presenting a criticism of prescriptive functions, the function of imparting mathematical knowledge and its boundaries are presented. A discussion of logic thinking, of alienation in modern societies, and of the functions of school mathematics in technocratic societies in general is presented that broadens the understanding of which social functions school mathematics might hold, leaving many open questions to explore.

RAISING THE QUESTION

Mathematics education is compulsory for all children in modern societies. What should be learnt and why it should be learnt are central questions of mathematics education research worldwide. Answers to these questions may vary in many aspects, but most mathematics education researchers agree on one point: routine calculations are over-represented and mathematics lessons should be more mind-challenging. Depending on perspective, »mind-challenging« may include focusing on creativity, problem solving, proofs and argumentations, applications and modelling, historical and social issues of mathematics, and so on; but, no matter the perspective, most suggest alternatives to routine calculations.[49]

The questions *why to teach* and *what to teach* are explicitly addressed throughout the mathematics education literature and in national standards documents. Heinrich Winter's (1995) essay "Mathematikunterricht und Allgemeinbildung"[50] and Hans-Werner Heymann's thesis (1996) *Allgemeinbildung und Mathematik*[51] are the most popular German examples of the first; the *Principles and Standards for School Mathematics* (2000) – published by the National Council of Teachers of Mathematics in the USA – and the *Bildungsstandards im Fach Mathematik* (2004) –

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[49] This paper will not discuss the corresponding concepts and show in what way they criticise too much routine calculations. For the purpose of this paper, it suffices to see that mathematics lessons comprehend routine calculations and that alternative concepts, in advocating different activities, are therefore directed against routine calculations. However, this is not to mean that mathematic educators or even mathematic teachers condemn routine calculations altogether.

[50] *Allgemeinbildung*, literally translated meaning *general education*, is a highly influential concept in German pedagogy without any conceptual equivalent in the English speaking world.

[51] Available to English readers as Heymann (2003) *Why Teach Mathematics? A Focus on General Education*.

published by the German Standing Conference of the Ministers of Education and Cultural Affairs and influenced by Heymann's work – are exemplars of the later.

Addressing what should be done implies that what is being done is not satisfactory. Winter and Heymann certainly belong to those who criticise too much calculating. But while the prescriptive concepts of Winter and Heymann suggest what should be done, they are unable to explain what is being done. Therefore, I ask: Is there any sense in having children master the masses of routine calculations, which we – the community of mathematics education researchers – might regard as being over-represented? Are there any reasons for the contemporary state of school mathematics? Can these explain why the abovementioned »mind-challenging« alternatives are not implemented on a large scale? And would not the answers to these questions strongly influence our ideas of what school mathematics should be like?

Pointing to tradition does not help here. Tradition may show us how the situation came to be, but it does not explain why some things changed while others have not. Therefore, I propose to address the issue in a broader context. Identifying school mathematics as an organ in our interacting, organic society, I raise the question: What are the social functions of school mathematics?

DISCUSSING NAÏVE ANSWERS

Curricular research offers a division between *material education*, i. e. mainly imparting knowledge, on the one hand and *formal education* on the other hand. My thesis is that the role of imparting mathematical knowledge is over-estimated and that further functions must be analysed to develop a comprehensive understanding of school mathematics.

A first approach to these functions might be a sceptical discussion of the concept of *competences* which curricular standards often use. These competences may already point to social functions of school mathematics, but starting with them creates problems. First, it is yet unclear how (or if) the demanded competences are indeed learned by children – especially when curricular concepts are used as tools for curricular reforms as is the case in the *Bildungsstandards im Fach Mathematik –*, and how (or if) these competences are indeed used outside school. Second, the focus on these competences might mask other functions of school mathematics that could be more central. Therefore, I suggest a different approach: After a discussion of the boundaries of imparting mathematical knowledge, I will elaborate on insightful connections between mathematical education on the one hand and Aristotelian logic, alienation and technocracy on the other hand. These are the points my studies concentrated on so far. In each case we can ask: What is the social impact of these and how does mathematical education contribute to them?

MATHEMATICAL KNOWLEDGE AND ITS BOUNDARIES

A function of school mathematics could be to have children impart certain mathematical knowledge in order to master certain situations that arise in society. Central questions are: What are these situations? What knowledge suits them? Is this knowledge indeed acquired in school?

A first set of situations that can be mastered mathematically is located in *private life*. Popular examples are cooking, shopping and trading, investments, or painting walls. A second set could consist of situations from *work life* that are not mathematics-intensive. Assuming that the mathematical knowledge required to master these situations is acquired in school mathematics, we nevertheless have to admit that the better part of school mathematics used in these situations has been taught after 7 or 8 years of school. For most people, quadratic, exponential, and trigonometric functions are not tools needed in mastering everyday situations in private or work life; neither are linear equation systems, calculus, conditional probability, and so forth. Heymann (2003, p. 104) argues:

> In their private and professional everyday lives, adults who are not involved in mathematics-intensive careers make use of relatively little mathematics. Everything beyond the content of what is normally taught up to 7th grade (computing percentages, computing interest rates, rule of three) is practically insignificant in later life.[52]

After comparing several studies exploring the uses of mathematics in private and work life, Heymann outlines the mathematical concepts that are frequently used (2003, pp. 88-89):

> *Arithmetic:* counting; mastery of basic arithmetical operations ('in one's head' or with paper and pencil, depending on the complexity); calculating with quantities, knowledge of the most important units of measurement, making simple measurements (primarily of time and distance); calculating fractions with simple denominators in unambiguous contexts; calculating decimal fractions; computing averages (arithmetic mean); computing percentages; computing interest rates; using the rule of three; completing arithmetical operations with a pocket calculator; basic skills in estimating and making rough calculations.
> *Geometry:* familiarity with elementary regular figures (circle, rectangle, square, etc.) and objects, as well as with elementary geometrical relationships and properties (perpendicularity, parallelism); ability to interpret and draw simple graphic representations of quantities and their relationships (charts, diagrams, maps) and the relationships between given points using Cartesian coordinate systems.

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[52] Heymann's thesis was followed by a vivid public discussion when the German red-top newspaper *Bild* (1995), disregarding the context of Heymann's work, printing the title »Professor: Too Much Maths is Nonsense« and stated, that »The mathematics adults need has been learned after 7 years of school.«

Mathematics certainly *is* used to master situations in private and work life, and this mathematics *is* being taught in school. Here, I must make two points, however. First, only until the seventh year of school, imparting mathematical knowledge can be regarded as a social function for people in non-mathematical jobs. Second, it is yet unclear whether the mathematical knowledge used in private and work life is indeed learned in school.

It is at least doubtful whether the mathematical knowledge used in private and work life is indeed learned in school. In her influential publication *Cognition in practice*, Jean Lave (1988) presents studies of the everyday use of mathematics in private and work life conducted in Liberia and the USA. She doubts whether »schooling is a font of transferable abilities« (p. xiii) and develops her thesis that mathematical knowledge needed to master situations in private and work life is learned »in practice« rather than in school. More recent studies, for example studies on the numeracy of nurses in the UK (see Coben, 2010, p. 14), support Lave's thesis. Heymann shares this view (2003, p. 98):

> A number of factors indicate that specific vocational mathematical qualifications tend to be learned more implicitly on the job and that thus the persons involved often remain unaware of them.

The thoughts presented above leave only a relatively small group of people engaged in mathematics-intensive professions, for whom higher mathematical qualification in school might be useful. Interpreted from a social perspective, it is possible that a function of school mathematics is to prepare as many children as possible for mathematics-intensive professions. It then would be reasonable to teach all children mathematics beyond their seventh year of school, attempting to maximise the number of children entering mathematics-intensive professions.

ENCULTURATION BEYOND KNOWLEDGE

Imparting mathematical knowledge is not as dominant a social function of school mathematics as might be expected. A critical examination of the sets of competences that curricular standards want children to learn in school mathematics suggests that competences such as »solving problems«, »modelling«, »using formal aspects of mathematics« or even »thinking logically« point at the nature of our engagement with the world, of our thinking. Our worldviews and the nature of our thinking depend on the society (or culture) in which we learned thinking and perceiving the world. This learning process can be called *enculturation*[53].

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[53] »Enculturation« as a terminology, derives from cultural studies. Sociologists prefer to speak about »socialisation« while educators often prefer »education«, although the latter has a strong intentional meaning.

Imparting mathematical knowledge is a part of enculturation, as it enables and encourages people to perceive and approach the world in a certain way, namely mathematically. Unfortunately, there is little literature on enculturation in school mathematics, though the work of, e.g., Roland Fischer, Alan Bishop and Ole Skovsmose is well acknowledged. However, Fischer's work (e. g. 2006) is very fragmentary and does not draw a comprehensive picture. Bishop's (1988) chapters on the »Values of Mathematical Culture« and on »Mathematical Culture and the Child« are highly important for my issue, but I do not want to discuss them in this paper. Skovsmose's work (2005) raises inspiring questions about the social functions of school mathematics, but he does not come to convincing answers.

Further related work are the analyses (and criticism) of »rationalism« and the role of mathematics in modern society in the work of Max Weber (1921/2008), as well as Max Horkheimer and Theodor W. Adorno (1944/1997), the sociologic analysis of mathematicians in practice by Bettina Heintz (2000) and the critical study concerning the legitimacy of modern mathematics by Philipp Ullmann (2008). Horkheimer and Adorno analyse and criticise how the ideas of Enlightenment shape our thinking and organise our society. Ullmann's work is strongly based on that of Horkheimer and Adorno but lays more emphasis on mathematics and its applications in society. Heintz' studies come to the conclusion that the modern mathematician is characterised by his will to avoid contradictions and that he therefore chooses a method that is intended to avoid contradictions, namely the logical proof. However, her work hardly draws attention to the social and educational implications of her results.

I do not want to discuss or present this literature in any more detail. Instead, I present my current, ever-evolving thinking of further social functions of school mathematics.

HIERARCHY AND LOGIC

The first thought I offer reaches back into the depths of history on human culture, specifically, to the development of hierarchical thinking. Roland Fischer (2006, pp. 133–141), building on the work of the Austrian philosopher Gerhard Schwarz (2007), describes *hierarchy* as a certain system of relationships between people of a certain community. In the history of human culture, the genesis of hierarchies can be observed wherever a community transitioned from a nomadic "tribe" to a fixed "state". The organising principle for tribes was based on kinship, setting a limit to the growth of a community. So, on the one hand, social growth and the development of states were only possible where hierarchies were established. On the other hand, the idea of organising communities hierarchically could only spread because the developing state communities were successful.

Living in a society that is ordered hierarchically is a powerful everyday experience: People learn that for every (but some) person(s) there is a person who decides what to do and what not to do; that actions are either allowed or forbidden; and that everyone is held responsible for his obedience or disobedience, neglecting the situation that led to her or his actions.

Living in such a society leaves its marks not only on the principles of our everyday actions but also on the principles of our everyday thinking. The idea of hierarchy became an element of thinking, of perceiving the world. For example, today we consider it normal to think of a husky as a dog, of a dog as a mammal, and so on. In fact, the biological classification is totally hierarchical.

In ancient Greece, scholars became aware of the principles of the thinking invoked by hierarchies. The *logic* of Aristotle is a description and analysis of this *logical thinking*. He postulates, for example, that every statement is either *true* or *false*. This may mean: allowed or forbidden in thinking. Furthermore, for every (but some) statement(s) we have statements on the basis of which we can decide whether the first statement is true or false. Eventually, the truth or falsity of a statement depends on the system of logic only, neglecting any (e. g. everyday life) connotations the statement may have.

While logic thinking was already a topic of ancient Greece philosophy, it was not until the beginning of modernity that logical thinking became conventionalised as the only "right" thinking with mathematics as its purest manifestation. René Descartes, the French mathematician and philosopher, may be considered the founder of this modern rationalism. In his *Rules for the direction of the mind*, he states, that »arithmetic and geometry alone are free of any error of falsity or uncertainty«[54] (1629/1959, pp. 8–9) and

> that those who seek the right way to truth must not engage with any matter that does not allow them to obtain a certainty comparable to that of arithmetic and geometric proofs.

This kind of thinking features a worldview that relies on antagonisms, causalities, and pre-determined, static concepts. It has certainly helped mankind to increase its understanding and possibilities of handling the world, but at the same time, it has shaped our thinking and perceiving the world in a certain way, leaving black spots and possibly discrediting those who think differently.

Logical thinking is not without alternative; it is not the only way of making sense. This alternative becomes clear when we look at non-Western societies (see Bishop 1988 for examples) or acknowledge that people had indeed thought in tribal communities before hierarchies and logic thinking evolved. The relentless division

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[54] Translated into English from the German translation of the Latin original *Regulae ad directionem ingenii* from 1629 by D. K.

into true or false has even been criticised from within mathematics. For instance, at the beginning of the twentieth century, the Dutch mathematician L. E. J. Brouwer (1918) began to create mathematics without the assumption that every statement must be either true or false.[55]

Connecting my aforementioned (somewhat oversimplified) analysis of logic to school mathematics, we might ask: Is logical thinking represented here, *more* than in any other school subjects, maybe even *only* here? Does school mathematics prepare children to think and act in a logically thinking and acting society? Extending the connection more critically, we might ask: What worldview do we create by teaching the dominance of logical thinking? And eventually, what does it mean for children who develop alternatives to "our" logic?

MODERNITY AND ALIENATION

At the verge to modernity, industrialisation made everyday life change dramatically. The medieval man (or woman) was a peasant or a craftsman, subsisting on what he produced. Although committed to kin, church, and state, he was the sovereign of his everyday life, making nearly every decision, especially the economic ones, on his own. This personal freedom was lost when more and more people began working in factories, where they had to perform prescribed repetitious work at a prescribed time of the day without causing any problems that might interfere with the production of the factory.

But it would be short sighted to assign the qualities of obedience, punctuality, and reliability in doing repetitious work to the factory worker of early industrialism alone. Contemporary work life requires the same qualities, and the modern employee must be enculturated to think, feel and act accordingly. The essence of this performance, which can be named *alienation*[56], is that a person must not act according to his actual feelings and wishes. Alienation is necessary for cooperative work where the work of many depends on the cooperation and reliability of the individual.

Primary and middle schools that emerged at the time of industrialisation took over a function of enculturation, preparing children to endure the alienation necessary for factory work. School mathematics was included from the very beginning and might have a particular role in the process of alienation so typical for the modern man and woman: Do the command-like masses of mathematics exercises drill obedience (cf. Skovsmose 2005)? Does the lack of individualisation in the mathematics classroom – in the process of teaching as well as in the nature of the answers expected from

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[55] In Brouwer's logic, statements can be neither true nor false. But still, they cannot be both true and false.

[56] Alienation here is understood in a slightly broader sense than in Marxian terms.

children – represent the factory's disregard for individual concerns? And to return to the beginning of this essay: Do routine mathematics calculations serve a social function, e. g., developing the ability and willingness to perform repetitious routine tasks whose broader sense might not be understood and/or favoured?

MODERN GOVERNMENT AND TECHNOCRACY

Modern government[57] has often been interpreted under the term *technocracy*. Technocrats (i.e., scientific specialists of a certain domain) name and determine the urgent questions of our time, planning work, health systems, education, economy, and so forth. Considering the aforementioned discussion, we may register that technocracy features a certain way of thinking; that is, logical thinking, and requires that people perform in a predictable, alienated fashion. Specifically, a technocracy requires people who follow rules which are not set up by themselves but by experts. (i. e., the technocrats).

Technocratic decision-making depends a lot on mathematics. Mathematical models are used to describe, prescribe, and predict technical, economical, and social matters. For example, medical studies claim that the effect of a new medication is twice as high as the old, the 2% increase of the GDP shows that the economy is doing well, or income taxes must be raised because the costs of the health system exceed the budget by 2 billion Euros. We accept these decisions, although we do not fully understand the justifications used.

But technocracy is nothing *imposed* on people; it is *lived* by people. Technocracy requires people to trust in it and it requires technocrats. Concerning the issue of trust, we might ask: How do people come to trust in mathematical justifications? Do people consider mathematics especially trustworthy?[58] And if so, do they develop this trustworthy attitude in school mathematics?

Concerning the issue of technocrats, another function of school mathematics can be identified. School mathematics might not only practice logical thinking, it might also select those children able to thinking logically, allocating the special few to technocrat positions in society. Ole Skovsmose (2005), in his book *Travelling Trough Education: Uncertainty, Mathematics, Responsibility*, raises the corresponding questions (p. 11):

> Could it be that mathematics education in fact acts as one of the pillars of the technological society by preparing well that minority of students who are to become 'technicians', quite independent of the fact that a majority of students are left behind?

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[57] »Government« here means any form of decision-making that other people depend on, not only in the executive of a state.

[58] Here, I omit an excessive, yet illuminating, discussion about the certainty and legitimacy of mathematics (cf. Skovsmose, 2005; Ullmann, 2008).

Could it be that mathematics education operates as an efficient social apparatus for selection, precisely by leaving behind a large group of students as not being 'suitable' for any further and expensive technological education?

FINAL THOUGHTS

In this essay, I have argued that school mathematics might have the social function of *identifying*, *selecting*, and *allocating* children, as well as preparing children for the contemporary predominant society in terms of

- developing children's *mathematical knowledge*,
- shaping children's *thinking* towards a form we may call logical, and
- shaping children's *feeling* towards a form that supports technocracy and living in a society that requires alienation.

As the works cited suggest (e. g., Skovsmose, 2005), many of these points have been discussed in the literature. These discussions are limited and often isolated, and fail to draw a cohesive picture of the social functions of school mathematics. Moreover, the discussions are highly evaluative, especially when it comes to people who suffer from mathematics education and are interpreted as being suppressed by a reign of technocrats. Although our own feelings and ideals *are* important, I am afraid that a perspective that places emphasis on the ethics of mathematics education might silence possible explanations that are necessary for a comprehensive understanding of the social functions of school mathematics.

Moreover, in this essay, I raise more questions rather than provide answers. The purpose of the essay, however, was to only mark the trajectory of my research project. My project aims not only to determine probable answers to the questions raised but also to develop a cohesive understanding of the social functions of school mathematics. This project requires not only further research on the functions discussed but also the development of deeper understandings of the intellectual concepts on the basis of which functions of school mathematics might be discussed.

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